A dimensionless analysis of the shear stress ratio on an alluvial ridge

1. **Dimensionless Parameters Definition:**

- $\beta = \frac{H_r}{H_c}$ is the ratio of the ridge height (H_r) to the channel depth (H_c). It measures how tall the ridge is compared to the depth of the flow that erodes it. A larger β means that the ridge is higher and has more elevation difference with the channel. This increases the shear stress on the ridge, as more potential energy is converted into kinetic energy by gravity. A smaller β means that the ridge is lower and has less elevation difference with the channel. This decreases the shear stress on the ridge, as less potential energy is converted into kinetic energy by gravity.
- $\gamma = \frac{S_r}{S_c}$ is the ratio of the ridge slope (S_r) to the channel slope (S_c). It measures how steep the ridge is compared to the slope of the flow that erodes it. A larger γ means that the ridge is steeper and has more resistance to erosion. This increases the shear stress on the ridge, as more frictional force is generated between sediment particles and water molecules. A smaller γ means that the ridge is gentler and has less resistance to erosion. This decreases the shear stress on the ridge, as less frictional force is generated between sediment particles and water molecules.

2. **Shear Stress on the Channel (τ_c):**
   - $\tau_c = \rho g H_c S_c$: Standard expression for shear stress in the channel.
3. **Shear Stress on the Ridge (τ_r):**
   - Given the definitions of β and γ, we can express the height of the ridge as $H_r = \beta H_c$ and the slope of the ridge as $S_r = \gamma S_c$.
   - The shear stress on the ridge, considering its unique characteristics, should be derived from its own height and slope: $\tau_r = \rho g H_r S_r$.
4. **Corrected Derivation of τ_r:**
   - Substituting $H_r = \beta H_c$ and $S_r = \gamma S_c$ into the formula for τ_r, we get:
     - $\tau_r = \rho g(\beta H_c)(\gamma S_c)$

- Simplifying, $\tau_r = \rho g \beta \gamma H_c S_c$.

5. **Ratio of Shear Stresses ($\frac{\tau_r}{\tau_c}$):**
   - Substituting the expressions for τ_r and τ_c into the ratio, we find:
     - $\frac{\tau_r}{\tau_c} = \frac{\rho g \beta \gamma H_c S_c}{\rho g H_c S_c}$
     - Simplifying, $\frac{\tau_r}{\tau_c} = \beta\gamma$.
6. **Threshold for Avulsion Initiation (Λ):**
   - The condition for avulsion initiation can be expressed as the ratio of shear stresses meeting or exceeding a threshold value Λ:
     - $\beta\gamma \geq \Lambda$.
   - Here, Λ represents the critical condition for the stability of the alluvial ridge, indicating that the ridge can maintain its shape and size as long as the dimensionless parameters (β and γ) are constant, regardless of changes in external factors.

$\Lambda = \frac{\tau_r}{\tau_c}$ represents the critical condition for the stability of the alluvial ridge. A constant Λ means that the shear stress ratio is independent of the absolute values of the shear stresses, but only depends on the relative values of the dimensionless parameters. This assumption implies that the alluvial ridge can maintain its shape and size as long as the dimensionless parameters are constant, regardless of the changes in discharge, sediment supply, or other external factors. However, if any of the dimensionless parameters change due to natural or human-induced disturbances, then the shear stress ratio will deviate from the threshold value, and the alluvial ridge will either grow or shrink until a new equilibrium is reached.